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Spontaneous breaking of scale invariance in a $D = 3$ $U(N)$ model with Chern-Simons gauge fields

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ABSTRACT: We study spontaneous breaking of scale invariance in the large N limit of three dimensional $U(N)_\kappa$ Chern-Simons theories coupled to a scalar field in the fundamental representation. When a $\lambda_6(\phi^\dagger \cdot \phi)^3$ self interaction term is added to the action we find a massive phase at a certain critical value for a combination of the λ_6 and 't Hooft's $\lambda = N/\kappa$ couplings. This model attracted recent attention since at finite κ it contains a singlet sector which is conjectured to be dual to Vasiliev's higher spin gravity on AdS_4 . Our paper concentrates on the massive phase of the 3d boundary theory. We discuss the advantage of introducing masses in the boundary theory through spontaneous breaking of scale invariance.

KEYWORDS: AdS-CFT Correspondence, $1/N$ Expansion, Nonperturbative Effects, Renormalization Group

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Contents

1	Introduction	1
2	Chern-Simons gauge field coupled to a $U(N)$ scalar - light cone gauge	3
2.1	1PI in the massive phase with spontaneously broken scale invariance	4
2.2	The vertex in the massive phase at order λ	6
2.3	The vertex in the massive phase, “seagull” graph contributions	6
3	Correlations	8
3.1	Vertex and “bubble” graph at leading N	8
3.2	The exact full planar vertex and “bubble graph”	9
3.3	$\langle J_0 J_0 \rangle$ correlator and the dilaton	11
3.4	Explicit breaking of scale invariance and the pseudo-dilaton	12
3.5	The $\langle J_0 J_0 J_0 \rangle$ correlator and the three dilatons interaction	13
4	Summary and conclusions	15

1 Introduction

There is recently new interest in the phase structure of large N $O(N)$ and $U(N)$ symmetric theories with matter fields in the fundamental representation. This interest [1–4] focused on the conjectured *AdS/CFT* correspondence between the singlet sector of the $O(N)$ vector theory in $d=3$ space-time dimensions and Vasiliev’s higher spin gravity theory on AdS_4 [5, 6]. The conjecture has been also tested by the computation of correlation functions of the higher spin gauge theory [7–10]. The quantum completion of the gravitational side of this duality is not known. If only tree level is considered, one has in this case an *AdS/CFT* correspondence for which both sides are weakly coupled. The advantage, in this case, is obvious as a testing ground of *AdS/CFT* ideas using explicit derivations [11–13].

Recent attention was also given to the large N limit of $O(N)_\kappa$ and $U(N)_\kappa$ level κ Chern-Simons gauge theories, at $N, \kappa \rightarrow \infty$ with a fixed ’t Hooft coupling $\lambda = N/\kappa$, coupled to scalar and fermion matter fields in the fundamental representation [14]–[18]. Pure Chern-Simons gauge theories possess first and second class constraints resulting in the absence of propagating degrees of freedom. Thus only the matter fields in the fundamental representation provide the true canonical degrees of freedom in these theories.

Interacting scalars and fermions in $O(N)$ and $U(N)$ symmetric field theories at large N are well understood and their phase structure is known [20]. In the presence of the Chern-Simons gauge field the singlet sector is singled out and the system fits well into the *AdS/CFT* conjectured duality. Using large N methods and the convenience of the light-cone gauge for the Chern-Simons action, explicit calculations can be performed shedding

light on the *AdS/CFT* correspondence. The original $O(N)$ model of $g(\vec{\phi})^4$ in [4] was deformed in [14]–[18] to include a marginal interaction term $(\lambda_6/24\pi)(\vec{\phi})^6$. It was conjectured that the gravity dual of these 3d theories on the boundary is a parity broken version of Vasiliev’s higher spin theory [5, 6] on AdS_4 in the bulk with a parity breaking parameter θ which depends on the ’t Hooft coupling $\lambda = N/\kappa$. Supersymmetric extension of this idea was introduced in [16].

The conjectured higher spin symmetry is an approximate symmetry valid at large N . The massless conformal invariant phase was analysed either by a generalized Hubbard-Stratanovich method [14] or in perturbation theory [15].

The theory of scalars and fermions in the fundamental representation of $U(N)$ coupled to Chern-Simons gauge fields are dual to the same bulk gravity theory on AdS_4 . Thus it has also been conjectured that there is a duality between the boson and fermion theories [17]. Calculation of the thermal free energy in large N Chern-Simons field coupled to fermions and scalars further strengthened this duality [18]. The thermal free energy has been calculated also in [14].

Ref. [18] concentrated on conformal theories in their “regular” and “critical” phase on the $d=3$ boundary. It was noted however that in the case of massive fermion and scalars the thermal free energy can be also calculated and the duality between the two Chern-Simons matter theories of fermion and boson may still exist. In this case high spin symmetry with the bulk Vasiliev’s AdS_4 is lost since the boundary theory is not conformal.

It was also mentioned in ref. [18] that the introduction of masses through spontaneous breaking of scale invariance [21]–[24] is an alternative at large N . Though at finite N the breakdown of the scale symmetry is explicit it is of $\mathcal{O}(N^{-1})$, namely the same order by which the *AdS/CFT* correspondence is approximated in the above mentioned conjecture.

We find it therefore appealing to further analyse the Chern-Simons matter theory with masses introduced through spontaneous breaking of scale invariance which assures the introduction of masses into the theory but leaves the boundary $d=3$ theory conformal to $\mathcal{O}(N^{-1})$.

In this paper we are emphasizing the massive phase and spontaneous breaking of scale invariance that occurs in this model at a fixed combination of coupling constants, $\lambda^2 + \lambda_6/8\pi^2 = 4$. After a brief introduction in section 2 we calculate in subsection 2.1 the boson mass gap for a self interacting boson in the fundamental representation of $U(N)$ coupled at a Chern-Simons gauge field. In subsection 2.2 the vertex is calculated in the massive phase with spontaneously broken scale invariance. Section 3 is devoted to calculations of several correlation functions in the massive phase.

In subsection 3.1 we calculate the ladder sum of the vertex and the resulting two point correlation of the scalar vertices — the “bubble graph”. In subsection 3.2 the exact full planar vertex and “bubble graph” is calculated and the effective four point coupling λ_4^{eff} is defined. In 3.3 we calculate the two point correlation $\langle J_0 J_0 \rangle$ in the large N limit to all orders in ’t Hooft coupling $\lambda = N/\kappa$ and the effective Lagrangian of the dilaton is introduced. Subsection 3.4 is devoted to explicit breaking of scale invariance and the pseudo-dilaton. In subsection 3.5 the three point correlation is calculated and the dilaton self- interaction is revealed. Summary and conclusions are found in section 4.

2 Chern-Simons gauge field coupled to a $U(N)$ scalar - light cone gauge

We will consider a complex scalar field $\phi(x)$ in the fundamental representation of $U(N)$ in three Euclidean dimensions coupled to a Chern-Simons level κ gauge field $A_\mu(x)$. To the free Chern-Simons action

$$\mathcal{S}_{\text{CS}}(\mathbf{A}) = -\frac{i\kappa}{4\pi}\epsilon_{\mu\nu\rho}\int d^3x \text{Tr}\left[\mathbf{A}_\mu(x)\partial_\nu\mathbf{A}_\rho(x) + \frac{2}{3}\mathbf{A}_\mu(x)\mathbf{A}_\nu(x)\mathbf{A}_\rho(x)\right], \quad (2.1)$$

we add the $U(N)$ invariant action

$$\mathcal{S}_{\text{Scalar}} = \int d^3x \left[(\mathbf{D}_\mu\phi(x))^\dagger \cdot \mathbf{D}_\mu\phi(x) + NV(\phi(x)^\dagger \cdot \phi(x)/N) \right], \quad (2.2)$$

where the complex $\phi(x)$ field is in the fundamental representation and \mathbf{D}_μ is the covariant derivative $\mathbf{D}_\mu = \partial_\mu + \mathbf{A}_\mu$. Here, $x^\mu = \{x^1, x^2, x^3\}$; $x^\pm = \frac{1}{\sqrt{2}}(x^1 \pm ix^2)$ and $\mathbf{D}_\pm = \partial_\pm + \mathbf{A}_\pm$. The contributions involving the interaction of the scalar and the gauge fields are generated by the covariant derivatives

$$\begin{aligned} \mathbf{D}_\mu\phi^\dagger \cdot \mathbf{D}_\mu\phi &= \partial_3\phi^\dagger \cdot \partial_3\phi + \partial_+\phi^\dagger \cdot \partial_-\phi + \partial_-\phi^\dagger \cdot \partial_+\phi \\ &\quad - \phi^\dagger\mathbf{A}_-\partial_+\phi - \phi^\dagger\mathbf{A}_+\partial_-\phi - \phi^\dagger\mathbf{A}_3\partial_3\phi \\ &\quad + \partial_+\phi^\dagger\mathbf{A}_-\phi + \partial_-\phi^\dagger\mathbf{A}_+\phi + \partial_3\phi^\dagger\mathbf{A}_3\phi \\ &\quad - \phi^\dagger(\mathbf{A}_3\mathbf{A}_3 + \mathbf{A}_+\mathbf{A}_- + \mathbf{A}_-\mathbf{A}_+)\phi \end{aligned} \quad (2.3)$$

$\mathbf{A} = A^a T^a$, T^a are antihermitian, normalized by $\text{Tr}\{T^a T^b\} = -\frac{1}{2}\delta^{ab}$. In the light-cone gauge $\mathbf{A}_- = \frac{1}{\sqrt{2}}(\mathbf{A}_1 + i\mathbf{A}_2) = 0$ and thus

$$\begin{aligned} \mathcal{S}_{\text{CS}} + \mathcal{S}_{\text{Scalar}} &= \int d^3x \left\{ \frac{\kappa}{4\pi} A_+^a \partial_- A_3^a - \phi^\dagger (\partial_3^2 + 2\partial_+ \partial_-) \phi \right. \\ &\quad - \phi^\dagger A_+^a T^a \partial_- \phi + \partial_- \phi^\dagger A_+^a T^a \phi \\ &\quad - \phi^\dagger A_3 T^a \partial_3 \phi + \partial_3 \phi^\dagger A_3 T^a \phi \\ &\quad \left. - \phi^\dagger (A_3^a A_3^a T^a T^b) \phi + NV(\phi^\dagger \cdot \phi/N) \right\} \end{aligned} \quad (2.4)$$

Since in the light-cone gauge the action is linear in A_+^a we have simply:

$$-\frac{\kappa}{4\pi}\partial_- A_3^a = J_-^a = \phi^\dagger T^a \partial_- \phi - \partial_- \phi^\dagger T^a \phi \quad (2.5)$$

With proper boundary conditions $A_3^a(x^-, x^+, x^3)$ is given by

$$A_3^a(x^-, x^+, x^3) = \frac{2\pi}{\kappa} \int_{-\infty}^{\infty} dx'^- \text{sgn}(x'^- - x^-) J_-^a(x'^-, x^+, x^3)$$

or

$$A_3^a(p) = \left(\frac{2\pi}{\kappa} \right) \frac{2ip^+}{p^{+2} + \epsilon^2} J_-^a \quad \epsilon \rightarrow 0 \rightarrow \left(\frac{4\pi i}{\kappa} \right) \frac{1}{p^+} J_-^a \quad (2.6)$$

When inserted in the action, one finds the gauge field propagator as the principal part of:

$$G_{+3}(p) = -G_{3+}(p) = \frac{4\pi i}{\kappa} \frac{1}{p^+} = 4\pi i \frac{\lambda}{N} \frac{1}{p^+}$$

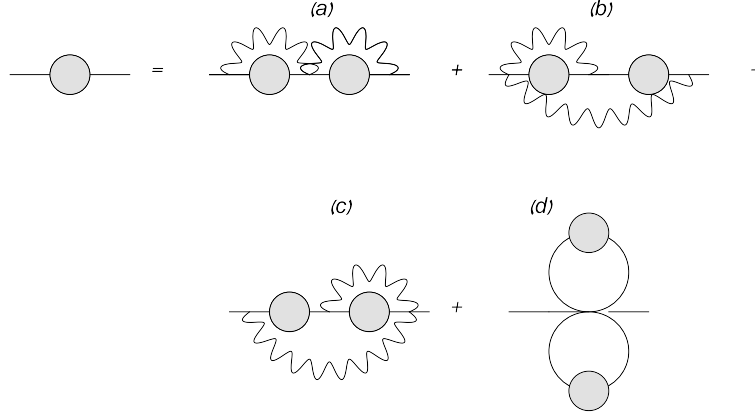


Figure 1. 1PI self energy graphs.

where the 't Hooft coupling is $\lambda = \frac{N}{\kappa}$

The following U(N) invariant self interacting scalar potential $V(\phi^\dagger \cdot \phi/N)$ will be considered now:

$$NV(\phi^\dagger \cdot \phi/N) = \mu^2 \phi^\dagger \cdot \phi + \frac{1}{2} \frac{\lambda_4}{N} (\phi^\dagger \cdot \phi)^2 + \frac{1}{6} \frac{\lambda_6}{N^2} (\phi^\dagger \cdot \phi)^3 \quad (2.7)$$

Following ref. [21], we will concentrate on the scale invariant potential with renormalized $\mu_R = \lambda_{4R} = 0$ and analyze the system in its massive phase of spontaneously broken conformal invariance.

2.1 1PI in the massive phase with spontaneously broken scale invariance

The one particle irreducible part of the scalar self-energy is given by the sum of diagrams in figure 1 [18]. The contributions of the diagrams [(a)+(b)+(c)] in figure 1 are:

$$\begin{aligned} \Sigma^{(a,b,c)}(p, \lambda)_{ij} = \delta_{ij} \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3l}{(2\pi)^3} \left\{ 4\pi^2 \lambda^2 \frac{(l+p)^+(q+p)^+}{(l-p)^+(q-p)^+} \left(\frac{1}{(q^2 + \Sigma(q))(l^2 + \Sigma(l))} \right) \right. \\ \left. - 8\pi^2 \lambda^2 \frac{(l+p)^+(q+l)^+}{(l-p)^+(q-l)^+} \left(\frac{1}{(q^2 + \Sigma(q))(l^2 + \Sigma(l))} \right) \right\} \end{aligned} \quad (2.8)$$

which sum up to

$$\Sigma^{(a,b,c)}(p, \lambda)_{ij} = 4\pi^2 \lambda^2 \delta_{ij} \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3l}{(2\pi)^3} \frac{1}{(q^2 + \Sigma(q))(l^2 + \Sigma(l))} \quad (2.9)$$

The sum of diagrams (a)+(b)+(c) in eq. (2.9) is of the same form as the contribution of diagram (d) of the scalar self interaction

$$\Sigma^{(d)}(p, \lambda)_{ij} = \frac{1}{2} \lambda_6 \delta_{ij} \int \frac{d^3q}{(2\pi)^3} \int \frac{d^3l}{(2\pi)^3} \frac{1}{(q^2 + \Sigma(q))(l^2 + \Sigma(l))} \quad (2.10)$$

It was shown that in the light-cone gauge $\Sigma(p)$ is a constant [14, 15] and thus the sum of eqs. (2.9) and (2.10) is given by:

$$\Sigma(p, \lambda, \lambda_6) = 4\pi^2 \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) \left\{ \int \frac{d^3q}{(2\pi)^3} \frac{1}{(q^2 + \Sigma(q))} \right\}^2$$

$$= 4\pi^2 \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) \left\{ \frac{1}{2\pi^2} \left(\Lambda - \frac{1}{2}\pi\sqrt{|\Sigma|} \right) \right\}^2 \quad (2.11)$$

A sharp UV cutoff is illustrated in eq. (2.11) as an example and any other UV regulator can be employed. The fully renormalized gap equation can be completed by adding to the gap equation the contributions of the $\mu^2 \phi^\dagger \cdot \phi$ and $\frac{1}{2} \frac{\lambda_4}{N} (\phi^\dagger \cdot \phi)^2$ terms from eq. (2.7). Namely, on the right hand side of eq. (2.11) will be added:

$$\lambda_4 \int \frac{d^3q}{(2\pi)^3} \frac{1}{(q^2 + \Sigma(q))} + \mu^2$$

The renormalized gap equation will be now:

$$\Sigma(p, \lambda, \mu, \lambda_4 \lambda_6) = \frac{1}{4} \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) |\Sigma| - \lambda_{4R} \frac{\sqrt{|\Sigma|}}{4\pi} + \mu_R^2 \quad (2.12)$$

In the conformal phase [21]

$$\mu_R = \lambda_{4R} = 0 \quad (2.13)$$

The renormalized 1PI self-energy is given by the solution of the gap equation:

$$\Sigma = \frac{1}{4} \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) |\Sigma| \quad (2.14)$$

Namely, there are two possible solutions:

(a) $\Sigma = M^2 = 0$ gives the conformal invariant massless phase discussed in refs. [14]–[17] or

(b) $\Sigma = M^2 \neq 0$ if $\lambda^2 + \frac{\lambda_6}{8\pi^2} = 4$ which results in a massive phase for this critical combination of the 't Hooft λ and self-interacting λ_6 couplings. This is a spontaneously broken scale invariance phase similar to the one encountered in ref. [21] in the case of the O(N) invariant self interacting $(\phi \cdot \phi)^3$ theory in three dimensions. This relation can be also written as

$$\lambda^2 + \frac{\lambda_6}{8\pi^2} = \lambda^2 + 4 \frac{\lambda_6}{\lambda_6^{crit}} = 4 \quad (2.15)$$

where $\lambda_6^{crit} = 32\pi^2$ is the critical value of the pure U(N) invariant case and is analogous to $\lambda_6^{crit} = 16\pi^2$ in the case of the pure O(N) invariant $(\phi \cdot \phi)^3$ case [21].

At the critical combination of the couplings in eq. (2.15) the massive phase is continually connected to the massless phase. This degeneracy, as seen also in the variational calculation in ref. [21], is due to a flat direction in the ground state energy. The degeneracy can be lifted by adding soft breaking terms and a unique ground state for either phase could be then created. A light pseudo-Goldstone particle should appear in the spectrum of the massive phase. One expects that the symmetric and broken phases will be degenerate in the conformal limit of the theory. Below the critical coupling the massless conformal phase is unique without the addition of soft breaking terms.



Figure 2. Vertex to order λ .

2.2 The vertex in the massive phase at order λ

The vertex in figure 2 at $k_+ = 0$ to first order in λ is given by:

$$\begin{aligned}
 V(p^2, k_3) &= 1 + \frac{4\pi i}{\kappa} \int \frac{d^3 l}{(2\pi)^3} \frac{1}{(l-p)^+} \\
 &\quad \{iT^a((p+l)^+(p+l+2k)^3 - (p+l)^3(p+l+2k)^+)iT^a\} \\
 &\quad \frac{1}{l^2 + \Sigma} \frac{1}{(l+k)^2 + \Sigma} \\
 &= 1 + i4\pi\lambda k_3 \int \frac{d^3 l}{(2\pi)^3} \frac{(l+p)^+}{(l-p)^+} \frac{1}{l^2 + \Sigma} \frac{1}{(l+k)^2 + \Sigma}
 \end{aligned} \tag{2.16}$$

After the angular integration in the 2d l plane

$$\int_0^{2\pi} \frac{d\phi_l}{2\pi} \frac{(l+p)^+}{(l-p)^+} = \Theta(l^2 - p^2) - \Theta(p^2 - l^2) = \epsilon(l^2 - p^2) \tag{2.17}$$

where l^2 and p^2 denote the variables in the two dimensional space, and after integrating also on l_3 , $V(p^2, k_3)$ is given by:

$$\begin{aligned}
 V(p^2, k_3) &= 1 + \frac{i\lambda k_3}{4} \int_0^\infty dl^2 \epsilon(l^2 - p^2) \int_0^1 dx (l^2 + x(1-x)k_3^2 + M^2)^{-3/2} \\
 &= 1 + \frac{i\lambda k_3}{2} \int_0^1 dx \{2(p^2 + x(1-x)k_3^2 + M^2)^{-1/2} - (x(1-x)k_3^2 + M^2)^{-1/2}\} \\
 &= 1 + i\lambda \left\{ 2 \arctan\left(\frac{k_3}{2\sqrt{p^2 + M^2}}\right) - \arctan\left(\frac{k_3}{2M}\right) \right\}
 \end{aligned} \tag{2.18}$$

One notes that at momentum transfer $k_+ = 0$ $V(p^2, k_3)$ depends only on the two dimensional vector \vec{p} and on the momentum transfer k_3 .

2.3 The vertex in the massive phase, “seagull” graph contributions

As seen in eqs. (2.9) and (2.10), the two contributions, λ^2 and λ_6 , result in similar contributions to the self-energy Σ . For the same reason when the vertex is calculated by adding an insertion at the appropriate lines in figure 1, one finds again that the contribution to the vertex of order λ^2 are of the same form as the order λ_6 contribution.

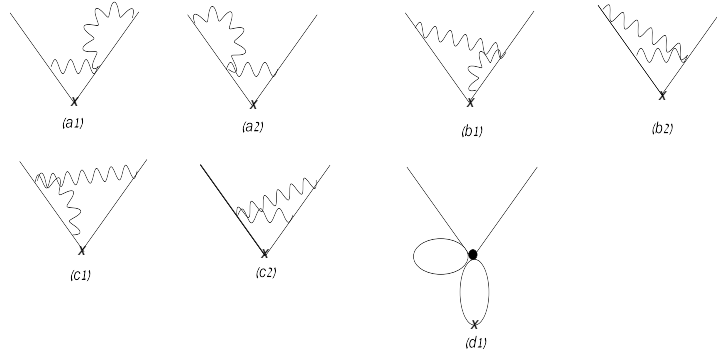


Figure 3. “Seagull” contributions to the vertex diagrams.

The contribution to the vertex of diagrams a1-2, b1-2 and c1-2 in figure 3 is:

$$\begin{aligned}
 V^{(a1-2,b1-2,c1-2)}(p,k) &= \lambda^2 4\pi^2 \int \frac{d^3 l}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \\
 &\left\{ -\left(\frac{l+p}{l-p}\right)^+ \left(\frac{q+p+k}{q-p}\right)^+ \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{(l+k)^2+\Sigma}\right) \left(\frac{1}{(q+k)^2+\Sigma}\right) \right. \\
 &\quad - \left(\frac{l+p}{l-p}\right)^+ \left(\frac{q+p+k}{q-p}\right)^+ \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{q^2+\Sigma}\right) \left(\frac{1}{(q+k)^2+\Sigma}\right) \\
 &\quad + \left(\frac{l+p}{l-p}\right)^+ \left(\frac{q+l+2k}{q-l}\right)^+ \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{(l+k)^2+\Sigma}\right) \left(\frac{1}{(q+k)^2+\Sigma}\right) \\
 &\quad + \left(\frac{l+p}{l-p}\right)^+ \left(\frac{l+q}{q-l}\right)^+ \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{q^2+\Sigma}\right) \left(\frac{1}{(q+k)^2+\Sigma}\right) \\
 &\quad + \left(\frac{l+q+k}{l-q}\right)^+ \left(\frac{q+p+2k}{q-p}\right)^+ \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{(l+k)^2+\Sigma}\right) \left(\frac{1}{(q+k)^2+\Sigma}\right) \\
 &\quad \left. + \left(\frac{l+q}{l-q}\right)^+ \left(\frac{q+p+k}{q-p}\right)^+ \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{q^2+\Sigma}\right) \left(\frac{1}{(q+k)^2+\Sigma}\right) \right\} \quad (2.19)
 \end{aligned}$$

The self interaction of the scalar fields contributes to the vertex the term

$$\begin{aligned}
 V^{(d1-2)}(p,k) &= -\frac{1}{2} \lambda_6 \int \frac{d^3 l}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \left\{ \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{(l+k)^2+\Sigma}\right) \left(\frac{1}{q^2+\Sigma}\right) \right. \\
 &\quad \left. + \left(\frac{1}{l^2+\Sigma}\right) \left(\frac{1}{q^2+\Sigma}\right) \left(\frac{1}{(q+k)^2+\Sigma}\right) \right\} \quad (2.20)
 \end{aligned}$$

When all vertex contributions are added at $k^+ = 0$, diagrams a1-2, b1-2, c1-2, d1-2 result in:

$$\begin{aligned}
 V^{(a-d)}(p,k_3) &= V = -8\pi^2 \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) \int \frac{d^3 l}{(2\pi)^3} \left(\frac{1}{l^2+\Sigma} \right) \int \frac{d^3 q}{(2\pi)^3} \\
 &\quad \left(\frac{1}{(l+k)^2+\Sigma} \right) \left(\frac{1}{q^2+\Sigma} \right) \quad (2.21)
 \end{aligned}$$

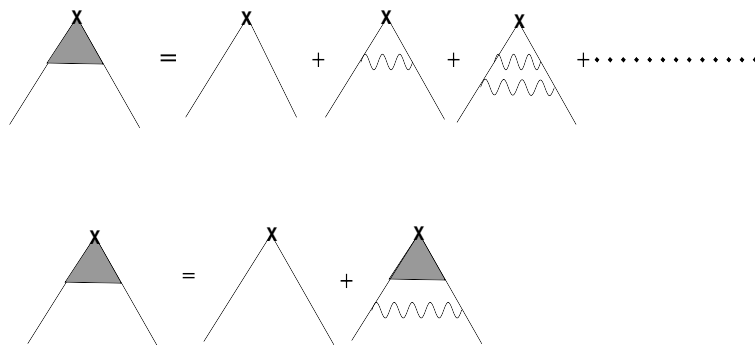


Figure 4. Vertex in the ladder approximation.

Namely, the combined contribution of order λ^2 and λ_6 results in a local vertex. The mutual cancelations which are presented in eqs. (2.11) and (2.21) is a general property of this model and occurs also in other amplitudes. We notice the coefficient $-8\pi^2(\lambda^2 + \frac{\lambda_6}{8\pi^2})$ equals $-32\pi^2$ at the critical coupling for the massive phase.

3 Correlations

3.1 Vertex and “bubble” graph at leading N

At large N and $k^+ = 0$, the vertex can be calculated to all orders in 't Hooft's coupling constant $\lambda = \frac{N}{\kappa}$. The order λ vertex corrections can be iterated, at $k^+ = 0$ by summing ladders to all orders as shown in figure 4.

The resummed vertex satisfies the following integral equation which can be explicitly evaluated

$$\begin{aligned}
 V(p^2, k_3) &= 1 + \frac{4\pi i}{\kappa} \int \frac{d^3 l}{(2\pi)^3} V(l^2, k_3) \frac{1}{(l-p)^+} \\
 &\quad \{iT^a((p+l)^+(p+l+2k)^3 - (p+l)^3(p+l+2k)^+)iT^a\} \\
 &\quad \frac{1}{l^2 + \Sigma} \frac{1}{(l+k)^2 + \Sigma} \\
 &= 1 + i4\pi\lambda k_3 \int \frac{d^3 l}{(2\pi)^3} V(l^2, k_3) \frac{(l+p)^+}{(l-p)^+} \frac{1}{l^2 + \Sigma} \frac{1}{(l+k)^2 + \Sigma} \quad (3.1)
 \end{aligned}$$

After the angle integration in the 2d l plane using eq. (2.17) and after integrating also on l_3 we have:

$$V(p^2, k_3) = 1 + \frac{i\lambda k_3}{4} \int_0^\infty dl^2 \epsilon(l^2 - p^2) V(l^2, k_3) \int_0^1 dx (l^2 + x(1-x)k_3^2 + M^2)^{-3/2} \quad (3.2)$$

where l^2 and p^2 denote the variables in the two dimensional transverse space. The general solution for $V(p^2, k_3)$ takes the form

$$V(p^2, k_3) = C \exp \left\{ i\lambda k_3 \int dx (p^2 + x(1-x)k_3^2 + M^2)^{-1/2} \right\} \quad (3.3)$$

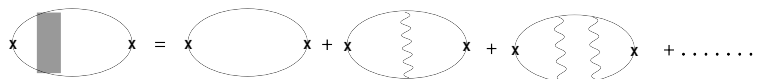


Figure 5. $B_{CS}(k_3)$ The “bubble graph” $B_{CS}(k_3)$.

and the constant C is found using eq. (3.2) and therefore satisfies.

$$C + V(p^2 = 0, k_3) = 2$$

Finally, we have for the vertex function

$$V(p^2, k_3) = 2 \exp \left\{ i\lambda k_3 \int_0^1 dx (p^2 + x(1-x)k_3^2 + M^2)^{-1/2} \right\} \left\{ 1 + \exp[i\lambda k_3 \int_0^1 (x(1-x)k_3^2 + M^2)^{-1/2}] \right\}^{-1} \quad (3.4)$$

The ladder contribution to the two point correlation of the scalar vertices can now be computed at $k^+ = 0$. We denote this by the “bubble graph” $B_{CS}(k_3)$ depicted in figure 5.

$$B_{CS}(k_3) = \int \frac{d^3 l}{(2\pi)^3} V(l^2, k_3) \left(\frac{1}{(l^2 + l_3^2 + \Sigma)} \right) \left(\frac{1}{l^2 + (l_3 + k_3)^2 + \Sigma} \right) \\ = \frac{1}{16\pi} \int dl^2 V(l^2, k_3) \int dx (l^2 + x(1-x)k_3^2 + \Sigma)^{-3/2} \quad (3.5)$$

Using the result for the vertex in eq. (3.4), the integral equation in eq. (3.5) gives the explicit expression for $B_{CS}(k_3)$:

$$B_{CS}(k_3) = \frac{1}{4\pi\lambda} \frac{1}{k_3} \tan \left\{ \frac{1}{2} \lambda k_3 \int dx (x(1-x)k_3^2 + M^2)^{-1/2} \right\} \\ = \frac{1}{4\pi\lambda} \frac{1}{k_3} \tan \left\{ \lambda \arctan \left(\frac{k_3}{2M} \right) \right\} \quad (3.6)$$

where $\Sigma = M^2$.

3.2 The exact full planar vertex and “bubble graph”

To compute the full planar scalar vertex, we need to be able to sum the contributions of the “seagull” graph exchanges combined with the ladder diagrams. At lowest order the “seagull” exchanges reduce to a bubble correction analogous to the insertion of the local λ_6 vertex contribution as in figure 3 (d) with the effect of simply renormalizing λ_6 by a λ^2 term. Ladder corrections to the “seagull” vertex are therefore the same as for a bare vertex.

The “seagull” corrections to the ladder vertex are more subtle as we must be able to perform the angular integrations that reduce the “seagull” exchange contributions to a local vertex as in section 2.3. Namely, one should be able to show that the local expression is obtained also when the lower vertex in figure 3 is not a constant but a function of the loop variables l_3 and l^2 , the two dimensional perpendicular variable. Indeed, by inspecting eqs. (2.19) one recognizes that this is clearly the case when $k^+ = 0$.

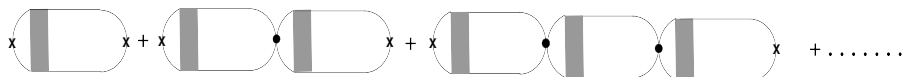


Figure 6. Full planar “bubble graph”.

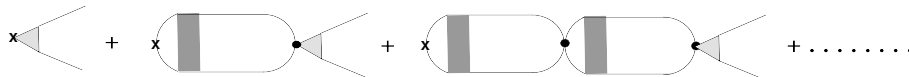


Figure 7. Full planar vertex.

(a) Integrating the angle in all terms in eq. (2.19) using eq. (2.17) and shifting the l_3 integration by k_3 when needed one notes that all the terms in front of the scalar propagators in eq. (2.19) sum up to a constant.

(b) Similarly, by inserting $k_+ = 0$ in eq. (2.19) and shifting the l_3 integration by k_3 when needed, one finds again that the six expressions in front of the scalar propagators add up to a constant,

In order to obtain the full scalar vertex we must sum to all orders iterations of the ladder and “seagull” exchanges. This results in a resummation of the “bubble” contributions depicted in figure 6 and figure 7. Hence, at leading order in the large N expansion, the scalar vertex at $k^+ = 0$ is given by

$$\begin{aligned}
 W(p^2, k_3) &= V(p^2, k_3) - B_{CS}(k_3)\lambda_4^{\text{eff}}V(p^2, k_3) \\
 &\quad + B_{CS}(k_3)\lambda_4^{\text{eff}}V(p^2, k_3)B_{CS}(k_3)\lambda_4^{\text{eff}}V(p^2, k_3) + \dots \\
 &= V(p^2, k_3) - B_{CS}(k_3)\lambda_4^{\text{eff}}W(p^2, k_3) \\
 &= V(p^2, k_3)(1 + \lambda_4^{\text{eff}}B_{CS}(k_3))^{-1}
 \end{aligned} \tag{3.7}$$

where

$$\lambda_4^{\text{eff}} = \lambda_4 + \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) \frac{8\pi^2}{N} \langle \phi^\dagger \cdot \phi \rangle \tag{3.8}$$

λ_4 is the self coupling $\frac{1}{2} \frac{\lambda_4}{N} (\phi^\dagger \cdot \phi)^2$ in eq. (2.7) and

$$\langle \phi^\dagger \cdot \phi \rangle = N \int \frac{d^3q}{(2\pi)^3} \frac{1}{q^2 + M^2} = N \left\{ \frac{\Lambda}{2\pi^2} - \frac{M}{4\pi} \right\} \tag{3.9}$$

In the absence of explicit breaking of conformal symmetry

$$\lambda_{4R} = \lambda_4 + 4\Lambda \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) = 0 \tag{3.10}$$

In the massless symmetric phase

$$\lambda_4^{\text{eff}} = 0$$

However, the effective coupling λ_4^{eff} in the massive spontaneously broken phase is:

$$\lambda_4^{\text{eff}} = \lambda_{4R} - 2\pi M \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) = -8\pi M \tag{3.11}$$

We summarize our results for the scalar vertex in the massive phase which are exact to all orders in the 't Hooft coupling, λ , and the ϕ^6 coupling, λ_6 , and to the leading order of the large N expansion. In this limit, the full scalar vertex is

$$W(p^2, k_3) = V(p^2, k_3)(1 - 8\pi M B_{CS}(k_3))^{-1} \quad (3.12)$$

where $V(p^2, k_3)$ is the ladder vertex of eq. (3.4) and $B_{CS}(k_3)$ is the bubble function given in eq. (3.6).

The above formulas reduce to those of ref. [17] in the massless limit, $M \rightarrow 0$, corresponding to the phase with explicit conformal symmetry.

3.3 $\langle J_0 J_0 \rangle$ correlator and the dilaton

The full correlator of two scalar currents at large N is now simply computed from the bubble integral with one bare vertex, one full vertex and the full scalar propagators. The resulting $\langle J_0(k) J_0(-k) \rangle$ correlator is analogous to the result found in pure $(\vec{\phi}^2)^3$ theory [21]

$$\langle J_0(k) J_0(-k) \rangle = N B_{CS}(k_3)(1 + \lambda_4^{\text{eff}} B_{CS}(k_3))^{-1} \quad (3.13)$$

The scalar currents are gauge invariant so we expect the scalar correlator to be a function of \vec{k}^2 although the explicit calculation was done with $k^+ = 0$.

The bubble summation, explicit in eq. (3.13), allows for the possibility of poles in the scalar correlator which would signify the existence of a composite dilaton. In the absence of explicit breaking of the conformal symmetry, the massive phase must contain a massless scalar dilaton.

The bubble function, $B_{CS}(k^2)$ in eq. (3.13) can be written as

$$\begin{aligned} B_{CS}(k) &= \frac{1}{8\pi M} + \frac{(\lambda^2 - 1)}{24\pi M} \left(\frac{k}{2M}\right)^2 \left\{ 1 + \left(\frac{k}{2M}\right)^2 \left(\frac{2\lambda^2 - 3}{5}\right) \right. \\ &\quad \left. + \left(\frac{k}{2M}\right)^4 \left(\frac{17\lambda^4 - 53\lambda^2 + 45}{105}\right) + \left(\frac{k}{2M}\right)^6 \left(\frac{62\lambda^6 - 295\lambda^4 + 503\lambda^2 - 315}{945}\right) \right. \\ &\quad \left. + \dots \right\} \\ &= \frac{1}{8\pi M} + \frac{(\lambda^2 - 1)}{24\pi M} \left(\frac{k}{2M}\right)^2 \sum_0^\infty \left(\frac{k}{2M}\right)^{2n} P_{2n}(\lambda) \end{aligned} \quad (3.14)$$

One notes that at $\lambda^2 = 1$ we have $B_{CS}(k) = 1/8\pi M$ for all values of k . Thus, in the conformal limit where $\lambda_4^{\text{eff}} = -8\pi M$ it is clearly seen that the pole at $\lambda^2 = 1$ in eq. (3.13) determines the physical boundary of λ .

Using the expansion in eq. (3.14), the low momentum behavior of the scalar correlator becomes

$$\begin{aligned} \langle J_0(k) J_0(-k) \rangle &= \frac{N}{8\pi M} \left\{ 1 - \frac{k^2}{12M^2} (1 - \lambda^2) + \dots \right\} \\ &\quad \left\{ 1 + \left(\frac{\lambda_4^{\text{eff}}}{8\pi M}\right) \left(1 - \frac{k^2}{12M^2} (1 - \lambda^2) + \dots \right) \right\}^{-1} \end{aligned} \quad (3.15)$$

At $\lambda_4^{\text{eff}} = -8\pi M$ the constant terms in the denominator cancel leaving the massless dilaton pole

$$\langle J_0(k) J_0(-k) \rangle = \frac{3N}{2\pi} \left(\frac{M}{1-\lambda^2} \right) \frac{1}{k^2} = \frac{f_D^2}{k^2} \quad (3.16)$$

where

$$f_D = \sqrt{\frac{3NM}{2\pi(1-\lambda^2)}} \quad (3.17)$$

Note the pole in f_D^2 at $\lambda^2 = 1$ which is, as mentioned, the boundary of physical couplings for λ .

From the expression for the full scalar vertex function the residue of the pole yields the coupling of the dilaton to scalar particles in the effective Lagrangian of the dilaton interactions. In terms of the dilaton field $D(x)$ (where $J_0(x) = f_D D(x)$) the effective Lagrangian is given by:

$$\mathcal{L} = \frac{1}{2} \partial_\mu D \cdot \partial_\mu D - g_D (\phi^\dagger \cdot \phi) D \quad (3.18)$$

where $g_D = -\frac{M^{3/2}}{\sqrt{N}} \sqrt{(96\pi)/(1-\lambda^2)}$ is determined from the infrared behavior of the scalar vertex in eq. (3.7)

$$W(p^2, k_3) = V(p^2, k_3) (1 + \lambda_4^{\text{eff}} B_{CS}(k_3))^{-1} \rightarrow \left(\frac{12M^2}{1-\lambda^2} \right) \frac{1}{k_3^2} = f_D g_D \frac{1}{k_3^2} \quad (3.19)$$

g_D^2 also has a pole as $\lambda^2 \rightarrow 1$. The boundary value of $\lambda^2 < 1$ implies a large positive value for λ_6 for the massive phase as the critical coupling condition in eq. (2.15) would require $\lambda^2 = 4$ without the λ_6 deformation.

3.4 Explicit breaking of scale invariance and the pseudo-dilaton

The massless dilaton in section 3.4 was found in the massive phase of the theory with spontaneously broken conformal invariance. This phase only exists at the critical coupling $\lambda^2 + \frac{\lambda_6}{8\pi^2} = 4$ in the conformal limit when $\mu_R^2 = 0$ and $\lambda_{4R} = 0$. However, we may also study the subcritical theory if we add terms which explicitly break the conformal symmetry but stabilize the vacuum of the massive phase. Without such symmetry breaking the subcritical theory has only the massless phase. If we are close to the critical coupling we can find solutions where the explicit breaking is small, the dilaton develops a small mass and the scalar bound state may be considered as a pseudo-dilaton.

We will consider now the massive phase of the near critical theory where the explicit breaking terms, μ_R^2 and λ_{4R} , are chosen so the gap equation in (2.12) has a stable solution with the mass of the ϕ boson equals M . We will expand solutions in λ_{4R} , μ_R^2 and in the deviation from criticality, $\delta \ll 1$, defined by

$$\frac{1}{4} \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) = 1 - \delta \quad (3.20)$$

The gap equation (2.12) takes the suggestive form

$$M^2 \delta = -\lambda_{4R} \left(\frac{M}{4\pi} \right) + \mu_R^2 \quad (3.21)$$

We can now explore the pole in the scalar current correlator in eq. (3.13) where $B_{CS}(k_3)$ is expanded as in eq. (3.14) and λ_4^{eff} is given in eq. (3.11)

$$\lambda_4^{\text{eff}} = \lambda_{4R} - 2\pi M \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) = \lambda_{4R} - 8\pi M(1 - \delta) \quad (3.22)$$

The expansion of the denominator at low momenta $k^2/M^2 \ll 1$ and $\lambda_{4R}/M, \delta \ll 1$ becomes

$$\begin{aligned} 1 + \lambda_4^{\text{eff}} B_{CS}(k) &= 1 + \{ \lambda_{4R} - 8\pi M(1 - \delta) \} \left(\frac{1}{8\pi} \right) \left\{ \frac{1}{M} - \left(\frac{1}{12} \right) \left(\frac{k^2}{M^3} \right) (1 - \lambda^2) \right\} + \dots \\ &= \left(\frac{\lambda_{4R}}{8\pi M} \right) + \delta + \left(\frac{1}{12} \right) \left(\frac{k^2}{M^2} \right) (1 - \lambda^2) + \dots \end{aligned} \quad (3.23)$$

From eq. (3.23) we can read off the mass of the pseudo-dilaton

$$M_{pD}^2 = \left(\frac{12M^2}{(1 - \lambda^2)} \right) \left[\left(\frac{\lambda_{4R}}{8\pi M} \right) + \delta \right] \quad (3.24)$$

For the expansion to make sense, the pseudo-dilaton mass must be small compared to the scalar boson mass, M . The two terms in eq. (3.24) can be of the same order as the explicit breaking by λ_{4R} can be small only if δ is also small.

For the near critical theory, the $U(N)$ singlet scalar bound state acts like a pseudo-dilaton. However, we also note the presence of a pole at $\lambda^2 = 1$ in the expressions for the dilaton mass in eq. (3.24). The limit $\lambda \rightarrow 1$ is thought to reflect the boundary of the Chern-Simons theory [15, 16] and the dilatonic interpretation of the scalar boson bound state is lost.

It may be interesting to consider the case when we leave $\mu_R = 0$ and take $\lambda_{4R} < 0$. The gap equation in eq. (2.12) has, in addition to the obvious massless solution, a massive solution with

$$\sqrt{|\Sigma|} = M = -\frac{1}{4\pi} \frac{\lambda_{4\pi}}{\delta} \quad (3.25)$$

The induced effective coupling in eq. (3.11) is now

$$\lambda_4^{\text{eff}} = \lambda_{4R} - 2\pi M \left(\lambda^2 + \frac{\lambda_6}{8\pi^2} \right) = -8\pi M \left(1 - \frac{1}{2}\delta \right) \quad (3.26)$$

We note that eq. (3.15) implies now a pseudo-dilaton solution with a mass given by

$$M_{pD}^2 = \delta \left(\frac{6M^2}{1 - \lambda^2} \right) \quad (3.27)$$

3.5 The $\langle J_0 J_0 J_0 \rangle$ correlator and the three dilatons interaction

Here we calculate the correlation function $\langle J_0(k) J_0(k') J_0(-k - k') \rangle$ in the massive ($M \neq 0$) phase again taking advantage of an expansion in k_3/M .

The first four diagrams (a,b,c,d) in figure 5 combine to give for the vertex:

$$V^{(a-d)}(k, k', -k - k') = \frac{N}{16\pi} \frac{1}{M^3} \left\{ 1 + \frac{(\lambda^2 - \frac{3}{2})}{12M^2} (k_3^2 + k_3'^2 + (k_3 + k_3')^2) + \mathcal{O}\left(\frac{(k_3, k_3')^4}{M^4}\right) \right\} \quad (3.28)$$

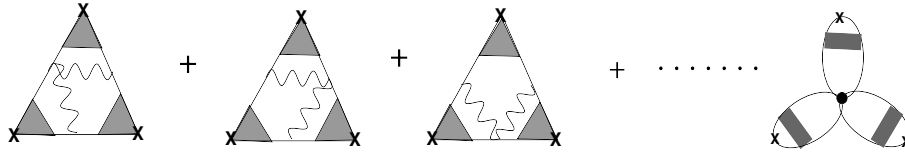


Figure 8. $\langle J_0(k) J_0(k') J_0(-k-k') \rangle$.

when diagram e is added one obtains in the same limit:

$$V_e = -\left(\lambda^2 + \frac{\lambda_6}{8\pi^2}\right) \frac{N}{64\pi} \frac{1}{M^3} \left\{ 1 + \frac{(\lambda^2 - 1)}{12M^2} (k_3^2 + k_3'^2 + (k_3 + k_3')^2) + \mathcal{O}\left(\frac{(k_3, k_3')^4}{M^4}\right) \right\} \quad (3.29)$$

In view of the gap equation in (2.14), the expansion in powers of k_3/M is valid only if in eqs. (3.28), (3.29) we choose $\lambda^2 + \frac{\lambda_6}{8\pi^2} = 4$ and thus $M^2 \neq 0$. At this critical point, the small k_3/M contribution of order λ^2 in (3.28) and (3.29) cancels out and we are left with

$$V_{\text{massive phase}} = -\frac{N}{16\pi} \frac{1}{24M^5} (k_3^2 + k_3'^2 + (k_3 + k_3')^2) + \mathcal{O}\left(\frac{(k_3, k_3')^4}{M^7}\right) \quad (3.30)$$

The full three current correlation function is obtained by adding the bubble iterations that depend on λ_4^{eff} to the vertex in eq. (3.30)

$$\begin{aligned} &\langle J_0(k) J_0(k') J_0(-k-k') \rangle \\ &= V(k, k', -k-k') (1 + \lambda_4^{\text{eff}} B_{CS}(k))^{-1} (1 + \lambda_4^{\text{eff}} B_{CS}(k'))^{-1} (1 + \lambda_4^{\text{eff}} B_{CS}(-k-k'))^{-1} \end{aligned} \quad (3.31)$$

Inserting here the above values of $V(k, k', -k-k')$, B_{CS} and λ_4^{eff} in the massive phase we find a simple result to leading order in the momentum expansion

$$\begin{aligned} &\langle J_0(k) J_0(k') J_0(-k-k') \rangle \\ &= -\frac{N}{384\pi} \frac{1}{M^5} (k_3^2 + k_3'^2 + (k_3 + k_3')^2) \left(\frac{k^2}{12M^2} (1 - \lambda^2) \right)^{-1} \\ &\quad \left(\frac{k'^2}{12M^2} (1 - \lambda^2) \right)^{-1} \left(\frac{(k+k')^2}{12M^2} (1 - \lambda^2) \right)^{-1} \\ &= -\frac{9N}{2\pi} \frac{M}{(1 - \lambda^2)^3} \left\{ \frac{1}{k^2 k'^2} + \frac{1}{k^2 (k+k')^2} + \frac{1}{k'^2 (k+k')^2} \right\} \\ &\quad + \mathcal{O}\left(\frac{M^3}{k^2}\right) \end{aligned} \quad (3.32)$$

From eq. (3.32) we can infer the three dilaton coupling. As the triple pole contribution cancels exactly, the nonderivative three dilaton coupling constant vanishes. We can interpret eq. (3.32) as evidence for a derivative interaction for the dilaton reflecting its nature as the Goldstone boson of spontaneously broken conformal symmetry.

The dilaton self interaction can be now defined in the effective Lagrangian

$$\mathcal{L}_3 D = g_{3D} (\partial_\mu D \cdot \partial_\mu D) D \quad (3.33)$$

One identifies in eq. (3.32) the three dilaton coupling g_{3D} from

$$f_D^3 g_{3D} = -N \frac{9}{2\pi} \frac{M}{(1 - \lambda^2)^3} \quad (3.34)$$

and using eq. (3.17) g_{3D} is given by

$$g_{3D} = -\sqrt{\frac{6\pi}{NM(1 - \lambda^2)^3}} \quad (3.35)$$

4 Summary and conclusions

We presented in this paper several aspects of spontaneous breaking of scale invariance in the large N limit of a three dimensional Chern-Simons theory coupled to a scalar field in the fundamental representation of $U(N)$. In the presence of self interaction term $\lambda_6(\phi^\dagger \cdot \phi)^3$, a massive phase is found at a critical value of the combination of λ_6 and 't Hooft's coupling, $\lambda = N/\kappa$, given by the condition $\lambda^2 + \lambda_6/8\pi^2 = 4$. At this critical value, conformal symmetry is spontaneously broken and a massless dilaton appears in the ground state.

Our explicit results are exact to leading order in the large N expansion and make use of the light-cone gauge for the Chern-Simons gauge field. At the critical coupling, we have shown that gap equation for the scalar self-energy has a massive solution in addition to the massless solution of the normal conformal phase. We have, then, computed the explicit solutions for the full scalar vertex function. Using these results, we are able to compute the exact scalar current correlation functions for the two point, $\langle JoJo \rangle$, and three point, $\langle JoJoJo \rangle$ correlators. These computations are exact in the large N limit to all orders in 't Hooft coupling, $\lambda = N/\kappa$, and the ϕ^6 coupling λ_6 that is, in turn, fixed by the criticality condition. From these correlation functions we have inferred the properties of the composite $U(N)$ singlet dilaton that arises in the theory due to the spontaneously broken scale symmetry in the massive phase. We have determined the effective dilaton decay constant, f_D , the coupling of the dilaton to the scalar boson, g_D and the self-coupling of the dilaton, g_{3D} . These couplings are all singular at $\lambda = 1$ which is thought to be the boundary value of the physical theory. Finally, we have also explored the near critical theory and found that there can exist phases with both spontaneous and explicit scale symmetry breaking. In these phases, the composite $U(N)$ singlet bound state is massive and plays the role of a pseudo-dilaton.

Chern-Simons gauge theories in three dimension have many remarkable properties which are only now beginning to be explored with the use of the large N expansion and other methods. As in the case of the massless conformal symmetric phase [14, 15, 19], also in the massive, spontaneously broken conformal invariance phase, the $U(N)$ singlet sector is conjectured to be dual to a parity broken version of Vasiliev's higher spin theory on AdS_4 in the bulk. It would be interesting to explore the implications for the bulk four dimensional AdS dual description of the massive phase. This is an open problem whose solution is not known at this point. In particular it is unknown whether the bulk theory is just a modification of Vassiliev's theory or whether new fields are required. The role

of the AdS dual of the composite dilaton behavior and meaning of the critical point for the specific combination of coupling constants would be interesting to explore. This could have implications for AdS duals of near conformal theories in four dimensions and, perhaps, the approximate AdS duals used to describe QCD in the large N limit. These interesting problems are beyond the scope of the present investigation.

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